# TD 2

16-17
L2 Math

## Ex 1

X taille d'un individu

$E(X) = 1,65 \qquad \sigma(X) = 0,05 \qquad V(X) = 0,05^2$

Markov

- $P(X \geq 1,8) \leq \frac{E(X)}{1,8} = \frac{1,65}{1,8} \simeq 0,917$

BT

- $P(X \geq 1,8) = P(X - 1,65 \geq 0,15)$

$$\leq P(|X - 1,65| \geq 0,15)$$

$$\leq \frac{0,05^2}{0,15^2} \simeq 0,111$$

## Ex 6

1) $S_n \sim \mathcal{B}(n, \frac{1}{2})$

2) $p = P(S_n > N \text{ ou } n - S_n > N)$

si $n \leq N \qquad p = 0$

si $n > 2N \qquad p = 1$

Supposons $N < n \leq 2N$

②

$$p = 1 - P(n-N \le S_n \le N)$$

$$= 1 - P\left( \frac{n/2 - N}{\sqrt{n/4}} \le \frac{S_n - n/2}{\sqrt{n/4}} \le \frac{N - n/2}{\sqrt{n/4}} \right)$$

$$= P\left( \left| \frac{S_n - n/2}{\sqrt{n/4}} \right| > \frac{N - n/2}{\sqrt{n/4}} \right)$$

$$\approx 2\left(1 - \Phi\left(\frac{N - n/2}{\sqrt{n/4}}\right)\right)$$

c) $n = 1000$ $\qquad 1 - \Phi\left(\frac{N-500}{\sqrt{250}}\right) \le 0{,}005$

$$\Phi\left(\frac{N-500}{\sqrt{250}}\right) \ge 0{,}995$$

$$\frac{N-500}{\sqrt{250}} \ge 2{,}58 \qquad N \ge \underbrace{2{,}58\sqrt{250} + 500}_{540{,}79}$$

$$N = 541$$

Ex 4

$E(X_i) = 1 \qquad \sigma(X_i) = \frac{1}{170}$

$X_1 \ldots X_n$ ⫫ m̂ loi

$S = \sum_{i=1}^{100} X_i \qquad P\left(\sum_{i=1}^{100} X_i > 100{,}15\right)$

$$P\left( \frac{\sum_{i=1}^{100} X_i - 100}{10 \times \frac{1}{170}} > \frac{100,15 - 100}{10/170} \right)$$

$$\approx 1 - \Phi(2,55) = 1 - 0,9946 = 0,0054.$$

Ex 7 . $Z_n \sim \mathcal{P}(n)$

$$Z_n = \sum_{i=1}^{n} X_i \qquad X_i \sim \mathcal{P}(1) \quad X_1 \dots X_n \perp\!\!\!\perp$$

$$E(Z_n) = n \qquad V(Z_n) = n$$

TLC $\Rightarrow$ $f_n(t) \longrightarrow \Phi(t)$

Ex 3

1) $P(X=k) \geq 0 \quad \forall\ k \in \{0, \dots, 9\}$

$$\sum_{k=0}^{9} P(X=k) = \sum_{k=0}^{9} \frac{10-k}{55} = \frac{100}{55} - \frac{1}{55} \sum_{k=0}^{9} k$$

$$= \frac{100}{55} - \frac{1}{55} \frac{9 \times 10}{2}$$

$$= \frac{100 - 45}{55} = 1$$

$$\underbrace{\sum_{k=0}^{n} k}_{\overset{\shortparallel}{3}} = \underbrace{\frac{n(n+1)}{2}}_{\frac{2 \times 3}{2} = 3.}$$

$n = 2$

2) $$E(X) = \sum_{k=0}^{9} k\, P(X=k) = \sum_{k=0}^{9} k \frac{10-k}{55}$$

$$= \frac{10}{55} \sum_{k=0}^{9} k - \frac{1}{55} \sum_{k=0}^{9} k^2$$

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

D'où $E(X) = \frac{10}{55}\,\frac{9\times10}{2} - \frac{1}{55}\,\frac{9\times10\times19}{6}$

$$= \frac{5\times2\times9\times10}{11\times5\times2} - \frac{1}{11\times5}\,\frac{3\times\cancel{3}\times2\times5\times19}{2\times\cancel{3}}$$

$$= \frac{5\times2\times9\times10 - 3\times2\times5\times19}{11\times5\times2} = \frac{900-570}{11\times5\times2} = \frac{330}{110}$$

$$= 3.$$

$$V(X) = E(X^2) - \left(E(X)\right)^2$$

$$E(X^2) = \sum_{k=0}^{9} k^2 P(X=k) = \sum_{k=0}^{9} k^2\,\frac{10-k}{55}$$

$$= \frac{10}{55}\sum_{k=0}^{9} k^2 - \frac{1}{55}\sum_{k=0}^{9} k^3$$

$$= \frac{10}{55}\,\frac{9\times10\times19}{6} - \frac{1}{55}\,\frac{9^2\,10^2}{4} \qquad \sum_{k=0}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$= \frac{10\times3\times\cancel{3}\times\cancel{2}\times\cancel{5}\times19}{\cancel{5}\times\cancel{2}\times\cancel{3}\times11} - \frac{9^2\times\cancel{2}\times5\times\cancel{2}\times\cancel{5}}{\cancel{5}\times\cancel{2}\times\cancel{2}\times11}$$

$$= \frac{10\times3\times19 - 9^2\times5}{11} = \frac{570-405}{11} = \frac{165}{11} = 15$$

$V(X) = 15 - 3^2 = 15 - 9 = 6$.

3) $\overline{X}_n$ nb moyenne de pièces deffectueuses produits par jour sur n jours

$E(\overline{X}_n) = 3$

$V(\overline{X}_n) = \frac{6}{n}$.

4) $S_n = \sum_{i=1}^{n} X_i$ = nb de pièces deffectueuses produits sur n jours

$n = 30$.

$P(S_n \leqslant 50)$

$E(S_n) = n \times E(X_i) = 3n$

$V(S_n) = n V(X_i) = n \times 6$

TLC

$\frac{S_n - n \times 3}{\sqrt{6n}}$ suit approx une loi $\mathcal{N}(0,1)$.

$n = 30$.

$\frac{S_{30} - 90}{\sqrt{180}}$ suit approx une loi $\mathcal{N}(0,1)$.

$\sqrt{180} = 6\sqrt{5}$.

$$P(S_{30} \leq 50) = P\left( \frac{S_{30}-90}{6\sqrt{5}} \leq \frac{50-90}{6\sqrt{5}} \right)$$

Soit $Z \sim \mathcal{N}(0,1)$

$$P(S_{30} \leq 50) = P\left( \frac{S_{30}-90}{6\sqrt{5}} \leq \frac{50-90}{6\sqrt{5}} \right)$$

$$\simeq P(Z \leq -2{,}98)$$

$$= 1 - P(Z \leq 2{,}98)$$

$$= 1 - 0{,}9986$$

$$= 0{,}0014.$$

Ex 5 appareil 500 places

$X_i = 1$ si le passager ne se présente pas
$\quad 0$ sinon

$X_i \sim \mathcal{B}(0,1)$

St $S_n = \sum_{i=1}^{n} X_i$ = nb de passagers qui ne se présentent pas

$n = 550$

$S_n \sim \mathcal{B}(550\,;\,0{,}1)$ $\quad n = 550$, $p = 0{,}1$

$$P(S_n < 50) = P\left(\frac{S_n - np}{\sqrt{np(1-p)}} < \frac{50 - np}{\sqrt{np(1-p)}}\right)$$

$$\approx P\left(Z < \frac{50 - 55}{\sqrt{550 \times 0{,}1 \times 0{,}9}}\right)$$

$$= P(Z < -0{,}71)$$

$$= 1 - P(Z < 0{,}71)$$

$$= 1 - 0{,}7611$$

$$= 0{,}2389$$